Q1a) Area of the gold band = πr^2

where $r = \frac{\text{diameter}}{2} = \frac{21}{2} = 10.5\text{cm}$.

$$A = \frac{22}{7} \times (10.5)^2$$

$$A = \frac{22}{7} \times 110.25$$

$$A = 346.5\text{cm}^2.$$

Radius of the Red = 10.5cm + 10.5cm = 21cm.
Radius of the Blue = 21cm + 10.5cm = 31.5cm
Radius of the Black = 31.5cm + 10.5cm = 42cm
Radius of the White = 42cm + 10.5cm = 52.5cm.

Hence:

Area of the red scoring region = Area of the red circle from the center − Area of the gold.

$$= \left(\frac{22}{7} \times (21)^2\right) - \left(346.5\text{cm}^2\right)$$

$$= \left(\frac{22}{7} \times 441\right) - \left(346.5\text{cm}^2\right)$$

$$= 1386\text{cm}^2 - 346.5\text{cm}^2$$

$$= 1{,}039.5\text{cm}^2$$

Also; Area of the Blue scoring region = Area of the blue circle from the centre − Area of the red circle from the centre.

$$= \left(\frac{22}{7} \times (31.5)^2\right) - \left(\frac{22}{7} \times (21)^2\right)$$

$$= \left(\frac{22}{7} \times 992.25\right) - \left(\frac{22}{7} \times 441\right)$$

$$= 3{,}118.5\,cm^2 - 1386\,cm^2$$

$$= 1732.5\,cm^2.$$

Area of the Black scoring region = Area of the black circle from the center − Area of the Blue circle from the center.

$$= \left(\frac{22}{7} \times (42)^2\right) - \left(\frac{22}{7} \times (31.5)^2\right).$$

$$= \left(\frac{22}{7} \times 1764\right) - \left(\frac{22}{7} \times 992.25\right)$$

$$= 5{,}544\,cm^2 - 3{,}118.5\,cm^2$$

$$= 2{,}425.5\,cm^2$$

Area of the white scoring region = Area of the white circle from the center - Area of the Black circle.

$$= \left(\frac{22}{7} \times (52.5)^2\right) - \left(\frac{22}{7} \times (42)^2\right)$$

$$= \left(\frac{22}{7} \times 2756.25\right) - \left(\frac{22}{7} \times 1764\right)$$

$$= 8662.5cm^2 - 5{,}544cm^2$$

$$= 3{,}118.5cm^2.$$

b) $12(x+6) = 5(y-2)$

$12x + 72 = 5y - 10$.

$12x - 5y + 72 + 10 = 0$

$12x - 5y + 82 = 0$.

Perpendicular distance =

$$= \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$$

where $a = 12, b = -5, c = 82,$
$x = -1, y = 1.$

$$= \frac{|12(-1) + (-5)(1) + (82)|}{\sqrt{(12)^2 + (-5)^2}}$$

$$= \frac{-12 - 5 + 82}{\sqrt{144 + 25}}$$

$$= \frac{65}{\sqrt{169}} = \frac{65}{13} = 5 \text{ units}.$$

Q2

a) Area of the square $= L^2$

$= (4)^2$

$= 16 cm^2$

Area of each quadrant :

$= \frac{1}{4} \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (1)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 1$

$= \frac{22}{28} = 0.7857 cm^2$

Hence the area of the four quadrants $= 4 \times 0.7857 cm^2$

$= 3.1428 cm^2$

Area of the circle in the square $= \pi r^2$ where $r = \frac{2}{2} = 1$.

$= \frac{22}{7} \times (1)^2$

$= \frac{22}{7} \times 1$

$= 3.1429 cm^2$

Therefore the area of the remaining portion :

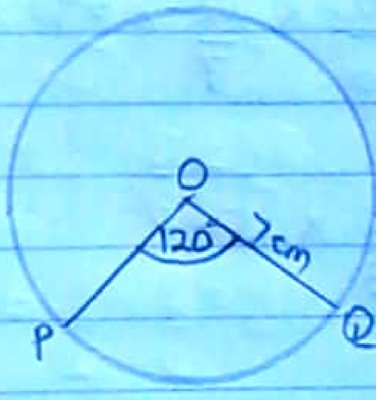
$16 cm^2 - (3.1428 cm^2 + 3.1429 cm^2)$

$16 cm^2 - 6.2857 cm^2$

$9.7143 cm^2$

$9.71 cm^2$ (2 d.p).

b)



$r = \frac{d}{2} = \frac{14 cm}{2} = 7cm$.

(i) $\frac{360° - \theta}{360°} \times 2\pi r$

$\frac{360° - 120°}{360°} \times 2 \times 3.142 \times 7$

$\frac{240}{360} \times 43.988$

1

Q2...

$\frac{2}{3} \times 43.988$

29.3253 cm.

The length of the major arc is 29.3253cm.

(ii) $\frac{360° - 120°}{360} \times 3.142 \times (7)^2$

$\frac{240}{360} \times \frac{22}{7} \times 49.$

$\frac{2}{3} \times 154$

$102.6667 cm^2$

The area of the major sector is $102.6667 cm^2$.

(iii) $2r + \frac{\theta}{360°} \times 2\pi r.$

where $\theta = 360° - 120° = 240°$

$2(7) + \frac{240}{360} \times 2 \times 3.142 \times 7$

$14 + \left(\frac{2}{3} \times 43.988\right)$

$14 + 29.3253$

43.3253 cm.

Thus the perimeter of the major ~~a~~ sector is 43.3253cm

Q3.

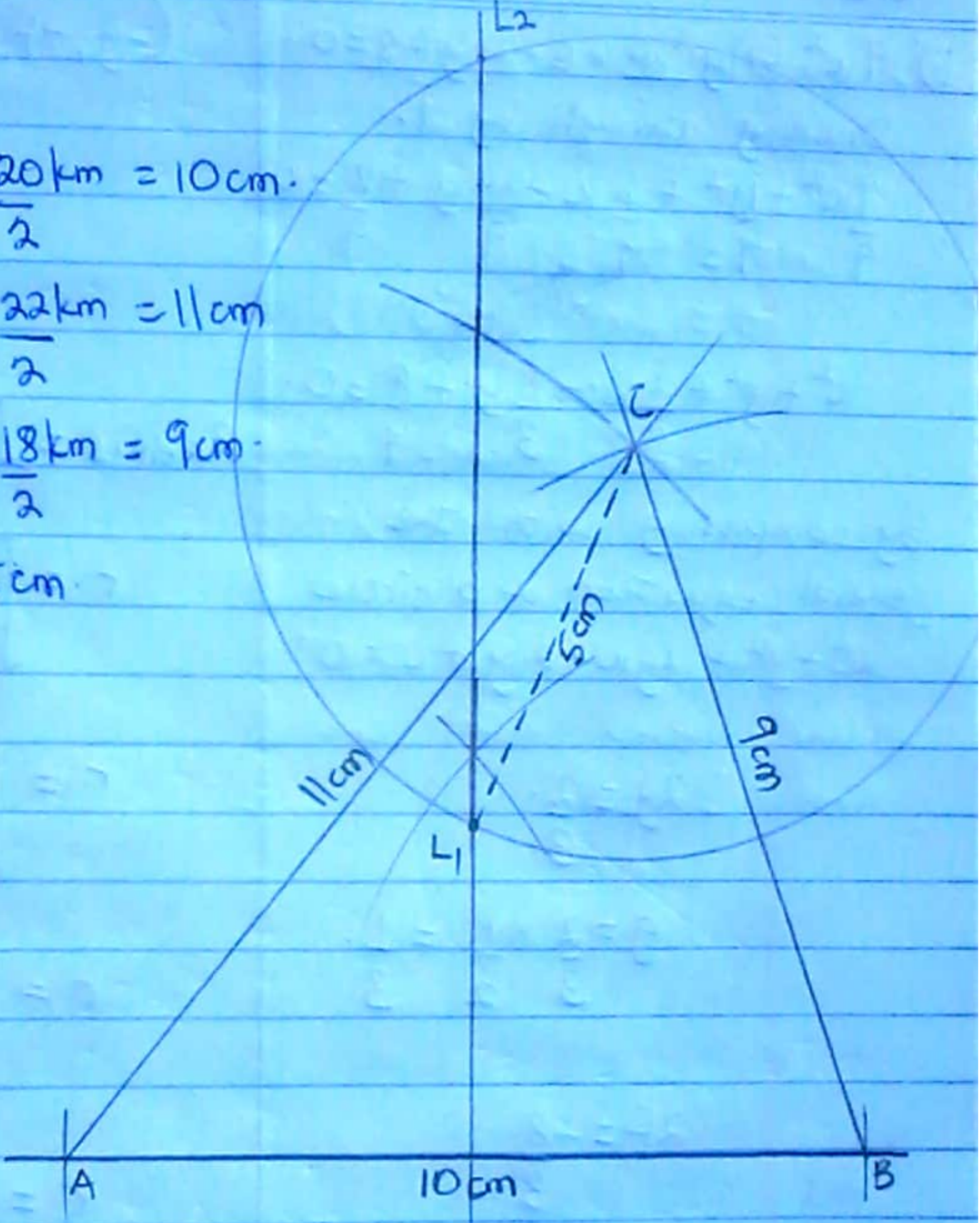
a) 1cm to 2km

$|AB| = 20km = \frac{20km}{2} = 10cm.$

$|AC| = 22km = \frac{22km}{2} = 11cm$

$|BC| = 18km = \frac{18km}{2} = 9cm.$

$|CL_1| = \frac{10km}{2} = 5cm.$



(i) 2 locations (L_1 and L_2)

(ii) From B to L_1 = 6.3cm = 12.6km

From B to L_2 = 14cm = 28km.

(iii) L_1, because it is located within the town and also near to the three towns, A, B and C.

Q3...

b) $9x^2 + 9y^2 + 6x - 24y + 8 = 0$.

Dividing through by 9.

$$\frac{9x^2}{9} + \frac{9y^2}{9} + \frac{6}{9}x - \frac{24}{9}y + \frac{8}{9} = 0$$

$$x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0.$$

By comparing with the general equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$;

$$2g = \frac{2}{3}.$$

$$g = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}.$$

$$2f = -\frac{8}{3}$$

$$f = -\frac{8}{3} \times \frac{1}{2}.$$

$$f = -\frac{8}{6} = -\frac{4}{3}.$$

Hence the centre of the circle is:

$$(-g, -f) = \left(-\tfrac{1}{3}, -\left(-\tfrac{4}{3}\right)\right)$$

$$= \left(-\frac{1}{3}, \frac{4}{3}\right).$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}.$$

$$r = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{-4}{3}\right)^2 - \frac{8}{9}}$$

$$r = \sqrt{\frac{1}{9} + \frac{16}{9} - \frac{8}{9}}$$

$$r = \sqrt{1}$$

$$r = 1\text{ cm}.$$

Thus the radius is 1cm.

Q4

a) (i) From $\triangle QPS$:

$\angle QPS + \angle PQS + \angle PSQ = 180^\circ$

(Angles in a triangle).

But $\angle PQS = \angle PSQ = 40^\circ$

So:

$\angle QPS + 40^\circ + 40^\circ = 180^\circ$

$\angle QPS + 80^\circ = 180^\circ$

$\angle QPS = 180^\circ - 80^\circ$

$\angle QPS = 100^\circ$

(ii) $\angle QRS + \angle QPS = 180^\circ$

(cyclic quadrilateral).

$\angle QRS + 100^\circ = 180^\circ$

$\angle QRS = 180^\circ - 100^\circ$

$\angle QRS = 80^\circ$

(iii) $\angle RQS + 40^\circ + 25^\circ + \angle PSQ = 180^\circ$

(Opp. angles in a cyclic quad.)

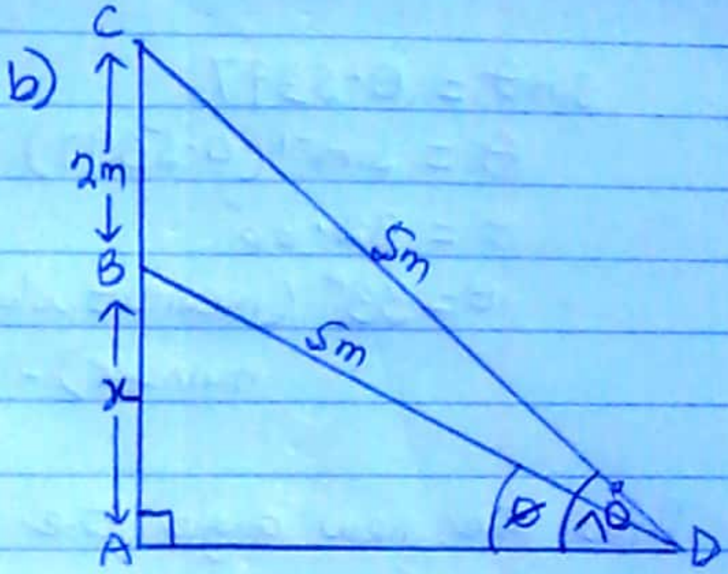
$\angle RQS + 40^\circ + 25^\circ + 40^\circ = 180^\circ$

$\angle RQS + 105^\circ = 180^\circ$

$\angle RQS = 180^\circ - 105^\circ$

$\angle RQS = 75^\circ$

b)



(ii) $\sin 70^\circ = \frac{|AC|}{5}$

$|AC| = 5 \times \sin 70^\circ$

$|AC| = 4.6985\text{m}$.

Hence:

$x = |AC| - 2\text{m}$.

$x = 4.6985\text{m} - 2\text{m}$

$x = 2.6985\text{m}$

$x = 2.70\text{m}$ (3 s.f).

The distance to the ground the ladder slipped from its initial position is 2.70m.

Scanned by CamScanner

Q4b.

(i) $\sin\theta = \frac{x}{5} = \frac{2.6985}{5}$

$\sin\theta = 0.5397.$

$\theta = \sin^{-1}(0.5397)$

$\theta = 32.66^\circ$

$\theta = 33^\circ$ (Nearest whole number).

Thus the new angle the Ladder makes with the ground is 33°.

Solved by:
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